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Linear and nonlinear dynamic model of a gantry crane system

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ABSTRACT – This paper investigates linear and nonlinear dynamic models for a Gantry Crane System (GCS). The system is a Single Input Multi Output (SIMO) system which a trolley displacement and payload oscillation as the outputs. The GCS is modeled by using the Lagrange Equation and both system responses are observed and analysed. The fundamental differences between the linear and nonlinear equations are presented. This analysis is beneficial for the development of efficient controllers for a GCS.

1. INTRODUCTION

A gantry crane system (GCS) is a machinery that is commonly used in industries for heavier materials (concrete, container, etc) that related to the process of transporting and carrying the load. GCS consists of a trolley and a payload is attached to the trolley by a cable vertically. The load with the cable is treated similar to a concept of a pendulum and free to oscillate in a 360 degree direction. For an analysis and development of controllers, the system has to be expressed by an accurate mathematical expression.

The development of mathematical concepts and techniques is aggressively implemented for all dynamic systems. The purpose is to represent the understanding of the actual phenomena system into theoretical for solving the problems that arise [1]. Normally, nature phenomena is always abounds with nonlinear systems due to material, inertia, body forces or friction [2]. However, this nonlinear system is quite difficult to be analysed due to the complex mathematical structure. Thus, nonlinear models can be linearized to a linear model in order to simplify the mathematical model to make it easier for controller designs.

2. MATHEMATICAL MODELING OF GCS

Several methods can be used to model the GCS. From several aspects of observations, the Lagrange's equation is more suitable and efficient to derive the mathematical expression, especially for a higher order system [3]. The GCS has two independent generalized coordinates, namely trolley displacement (x) and payload oscillation (θ). The structural model and all the parameter values of GCS are presented in Figure 1 and Table 1 respectively. Some assumptions have been made such as cable of trolley and hanged load are assumed to be rigid and massless.

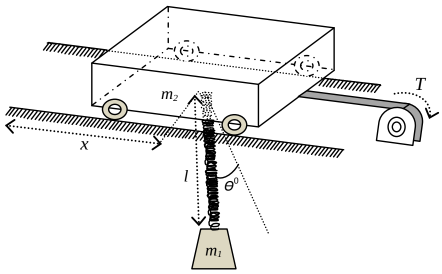

Figure 1 Structural model of GCS.

Table 1 System parameters [3].

Parameters	Values
Payload mass (m_1)	0.5 kg
Trolley mass (m_2)	2 kg
Cable length (l)	0.5 m
Gravitational (g)	9.81 m/s^2
Damping coefficient (B)	0.001 Ns/m
Resistance (R)	2.6 Ω
Torque constant (K_T)	0.007 Nm/A
Electric constant (K_E)	0.007 Vs/rad
Radius of pulley (r_P)	0.02 m
Gear ratio (z)	0.15

The standard form of Lagrange's equation is written as:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right]-\frac{\partial L}{\partial q_i}=Q_i \qquad (1)$$

Where L, Q_i and q_i represent the Lagrangian function, nonconservative generalized forces and independent generalized coordinates respectively. The Lagrangian function can be written as:

$$L=T-P \qquad (2)$$

Solving for Equation (1) and (2) yields differential equations as:

$$(m_1 + m_2)\ddot{x} + m_1 l\ddot{\theta}\cos\theta - m_1 l\dot{\theta}^2 \sin\theta + B\dot{x} = F \tag{3}$$

$$m_1 l^2\ddot{\theta} + m_1 l\ddot{x}\cos\theta + m_1 g l\sin\theta = 0 \tag{4}$$

Where F is external force. By considering the dynamic of a DC motor is included in this GCS model, a complete nonlinear equation of the GCS can be obtained as:

$$V = \left[\frac{RBr_P}{K_T z} + \frac{K_E z}{r_P}\right]\dot{x} + \left[\frac{Rr_P}{K_T z}\right](m_1 l)\left[\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right] + \left[\frac{Rr_P}{K_T z}\right](m_1 + m_2)\ddot{x} \tag{5}$$

$$m_1 l^2\ddot{\theta} + m_1 l\ddot{x}\cos\theta + m_1 g l\sin\theta = 0 \tag{6}$$

Where V is an input voltage and all the parameters are presented in Table 1. In order to linearize the nonlinear model, small θ (sin $\theta \approx \theta$ and cos $\theta = 1$) during system operation is considered. Therefore, the linear equations can be obtained as:

$$V = \left[\frac{RBr_P}{K_T z} + \frac{K_E z}{r_P}\right]\dot{x} + \left[\frac{Rr_P}{K_T z}\right](m_1 l\ddot{\theta}) + \left[\frac{Rr_P}{K_T z}\right](m_1 + m_2)\ddot{x} \qquad \text{`(7)}$$

$$l\ddot{\theta} + \ddot{x} + g\theta = 0 \tag{8}$$

3. RESULTS AND DISCUSSION

Figure 2 shows the Simulink model of linear and nonlinear models of GCS. The linear and nonlinear equations in Equation (5), (6), (7) and (8) were applied in the Simulink model and the system responses are observed and analysed for their similarity. Figures 3 and 4 show the trolley displacement and payload oscillation responses with a pulse input.

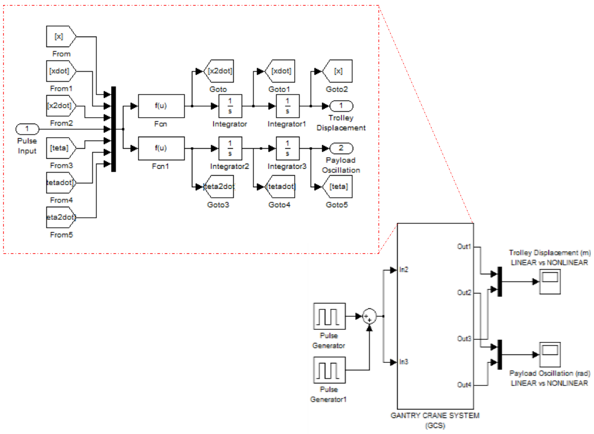

Figure 2 Simulink model of linear and nonlinear GCS.

It is noted that a very close agreement between the linear and nonlinear models have been obtained for both system responses (trolley displacement and payload oscillation). It can be further shown by the trolley displacement response in a selected time period as shown in Figure 3. It shows that, within a small angle, the linear model can represent a nonlinear model and provide a simpler equation for controller design.

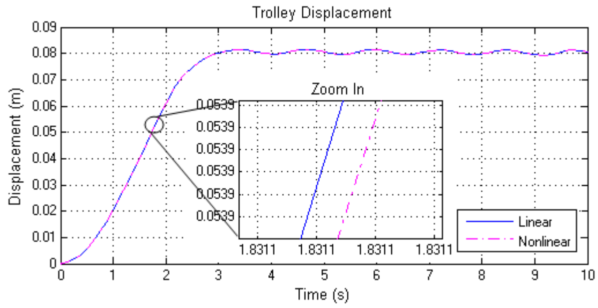

Figure 3 System response of trolley displacement.

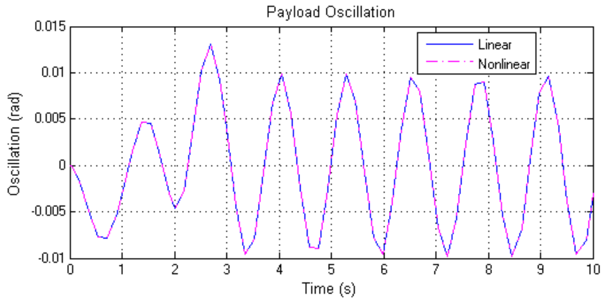

Figure 4 System response of payload oscillation.

4. CONCLUSION

Designing of controllers is easier by using a linear model as the derived mathematical models are less complicated. For the GCS, it has been proven that both linear responses (trolley displacement and payload oscillation) are in a good agreement with a natural response that present in a nonlinear system. Thus, it is reasonable to estimate or approximate all the nonlinear model parameters with the linear model.

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