Nonlinear behavior of a plate with an arbitrarily orientated crack

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ABSTRACT – This paper presents a nonlinear analysis for a thin isotropic plate containing an arbitrarily orientated surface crack. The governing equation of motion of the plate model with enhanced crack modelling and subjected to transverse harmonic excitation is proposed to represent the vibrational response of the plate and is based on classical plate theory into which a developed crack model has been assimilated. An approximate solution method based on the perturbation method of multiple scales is applied and the nonlinear behavior of the cracked plate model is investigated from the amplitude-frequency equation. It is found that the nonlinear characteristics of the cracked plate structure can be greatly affected by the orientation of the crack in the plate.

1. INTRODUCTION

A vibration analysis for a cracked thin isotropic plate has been motivated by the well-known applicability of various vibrational techniques for structural damage detection. Importantly, it is necessary still to develop a deep understanding of the derivation of the model of a cracked plate and it behaviours, especially for nonlinear case. Much research work has been undertaken on linear model, and there are restricted nonlinear models available for vibration problems in cracked plates.

A detailed derivation of the differential equation based on classical plate theory for modelling a crack in a plate for nonlinear model was first initiated by Israr et al. [1]. In these work, the concept of a line-spring model based on Kirchoff's plate bending theories, as first introduced by Rice and Levy [2], was used for the crack formation. Ismail and Cartmell [3] have provided an extension to the development of currently available analytical models of the vibration characteristics of a cracked plate structure, particularly for an alternative geometry in which the crack orientation is variable. Gangadhar et al. [4] presented an analytical model for nonlinear vibration analysis of a thin isotropic by considering two perpendicular, partial surface cracks located at the centre of the plate.

An approximate analytical method based on the perturbation method used in order to study and interpret the physical behaviour of a cracked plate. One of the most widely used perturbation methods is the method of multiple scales [5]. This is frequently used for obtaining close-form solution for nonlinear problems. The basic idea behind this approach is that the single independent variable, T is uniformly split up into several new independent variables for an example T_1, T_2, $T_3, \ldots T_n$ and these independent variables define successively slower dependencies for the dependent variables when expressed in term of a uniformly valid expansion equation. Hence, the aim of this paper is to investigate the nonlinear behaviour of the cracked plate model discussed in paper by [3] in which a plate containing an arbitrarily orientated surface crack is considered.

2. METHODOLOGY

The equation of motion for a plate containing a surface crack of variable angular orientation has been derived by Ismail and Cartmell [3]. Then, by applying the Berger formulation the derived governing equation of motion of this cracked plate is converted into a nonlinear ordinary differential equation model. This Berger formulation can be used to investigate nonlinear vibrations when the strain energy due to second invariant of the strains in the middle surface of the plate can justifiably by ignored. Finally, by considering the system to be under the influence of weak classical linear viscous damping, μ and the load to be harmonic, q leads to the form of a specialized Duffing equation as follows:

$$\ddot{\psi}_{ij}(t) + 2\mu\dot{\psi}_{ij}(t) + \omega_{ij}^2\psi_{ij}(t) + \gamma_{ij}\psi_{ij}^3(t) = \frac{\eta_{ij}}{D} q\cos\Omega_{ij}t \tag{1}$$

The explanation about each terms in this equation can be found in paper by [3]. Next, the first order multiple scales method is applied for obtain close-form solution in order to investigate the nonlinear behavior of this cracked plate model. So, for the coordinate ψ_{ij}, the dependent variables would typically have this form;

$$\psi_{ij}(t,\varepsilon) = \psi_{0ij}(T_0,T_1) + \varepsilon\psi_{1ij}(T_0,T_1) + o(\varepsilon^2) \tag{2}$$

Where $T_n = \varepsilon^n t$ and the parameter, ε, is known as a (small) perturbation parameter with $\varepsilon \ll 1$. ψ_{0ij} and ψ_{1ij} are solution functions yet to be determined and T_0 and T_1 are successively slower time scales. Before applying the method of multiple scales to obtain an approximate solution to this problem it is necessary to *order* the cubic term, the damping, and the excitation term. These terms are ordered by means of the small parameter ε according to their perceived relative numerical strength. To accomplish this we assume that the cubic, damping and the excitation terms are a definitionally weak term, thus these are assumed to become:

$$\gamma = \varepsilon \hat{\gamma}; \;\; \mu = \varepsilon \hat{\mu}; \text{ and } \; q = \varepsilon \hat{q} \tag{3}$$

Finally, uniformly valid expansion for the first order approximate solution can be obtained and after converting the exponent terms into trigonometric forms the full solution to first order ε becomes:

$$\psi_{ij}(t,\varepsilon) = b\cos(\Omega t - \phi) + \frac{\hat{\gamma}_{ij} b^3}{32\omega_{ij}^{\ 2}}\cos(3\Omega t - 3\phi) + o(\varepsilon^2) \tag{4}$$

Into which numerically calculated values for b can be obtained from the amplitude-frequency equation as follows:

$$\hat{\mu}^2 b^2 + \left(\sigma_{ij} - \frac{3\hat{\gamma}_{ij} b^2}{8\omega_{ij}}\right)^2 b^2 = \frac{\eta_{ij}^{\ 2}}{4D^2\omega_{ij}^{\ 2}}\hat{q}^2 \tag{5}$$

3. RESULTS AND DISCUSSION

The analytical results are shown in Figure 1.

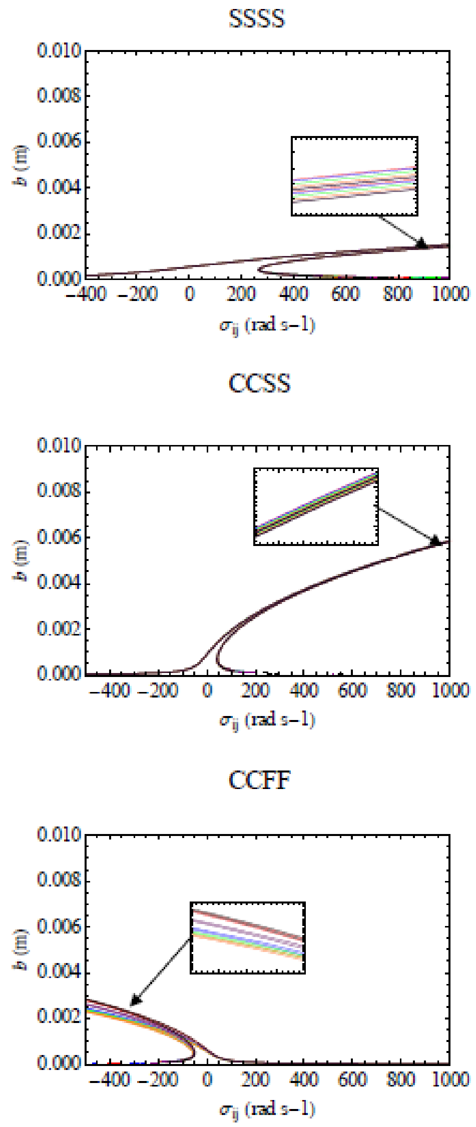

Figure 1 Nonlinear response curves for the cracked plate model for three types of boundary conditions (0°: Red line, 20°: Blue line, 40°: Green line, 60°: Orange line, 80°: Purple line, 90°: Black line).

This Figure show curves for the nonlinear response which represent the behavior of rectangular plate containing the orientated surface crack for the three different types of boundary conditions. Equation (5) is used to plot these curves and the aspect ratio chosen for the rectangular plate is 1:2. The system displays typical nonlinear characteristics, as evident in the Figure, with characteristic hardening and softening phenomena for a 0.003 m half-crack length. In this Figure, for the cracked plate model with the SSSS and CCSS boundary conditions, the nonlinearity bends the curves to the right, as for a hardening system. In this case the nonlinear hardening effect is clearly much stronger for the SSSS boundary condition. However, for the CCFF boundary condition the nonlinearity bends the curves to the left as for a softening system. The influence of the crack orientation angle on the frequency response is also observed. Cases tested for the SSSS and CCSS boundary conditions show no obvious hardening effects for rectangular plates. For the CCFF boundary condition it can clearly be seen that the nonlinear hardening effect increases up to 60° and then reduces when the crack orientation angles starts to exceed 60°. It should be noted that the amplitude decreases with the increase in frequency.

4. CONCLUSIONS

The nonlinear behavior of the cracked plate model has been investigated from the amplitude-frequency equation and this has showed that the inclusion of a crack within the plate produces a global effect on the nonlinear response of the overall system. As a conclusion, it can be said that the nonlinear characteristics of the plate structure are affected by the orientation of the crack.

5. REFERENCES

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