

# The influence of boundary condition on the deformed shape of axially compressed cones

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**ABSTRACT** – The aim of this research work is to investigate the influence of boundary condition on the buckling load and deformed shape of axially compressed cone. Two boundary conditions are considered, they are: (i) fully clamped at the bottom and allowed to move in the axial direction at the top ends, and (ii) allow all displacement movement except axial direction at the bottom and restrict all movement at the top except axial direction. Cones were relatively thick with nominal wall thickness of 2 mm and the semi-vertex angle,  $\beta$  is  $15^\circ$ , hence their buckling behavior remains within the elastic-plastic range. The geometry of radius of bigger radius,  $r_2$ , to radius of smaller radius,  $r_1$ , ( $r_2/r_1 = 1.75$ ), and the ratio of radius-to-thickness ( $r_2/t$ ) was taken as 35. Results indicate that the change of boundary condition has a strong influence on both the buckling load and the deformed shape.

radius,  $r_2$ , to radius of smaller radius,  $r_1$ , ( $r_2/r_1 = 1.75$ ), the ratio of radius-to-thickness ( $r_2/t$ ) was taken as 35 and semi-vertex angle of  $15^\circ$  degrees. Cones are assumed to be made from mild steel with material properties presented in Table 1. Two different boundary conditions were employed in this present work as given in Table 2. The notation used above assumes:  $u \equiv$  displacements, and  $\Phi \equiv$  rotations.

Table 1 Set of mechanical properties of A36 steel material obtained from uni-axial testing.

Sample	E (GPa)	Upper yield (MPa)	UTS (MPa)
1	219.219	367.1	403.6
2	238.253	371.1	409.9
3	201.993	371.6	411.2
Average	219.822	369.9	408.2

## 1. INTRODUCTION

Conical shells structures primarily used in engineering application are susceptible to imperfection. One of the major source of imperfection commonly encountered in the industry is the influence of boundary condition (edge support) when such structures are used. Very few studies have been carried out on the influence of boundary condition on the load carrying capacity of cones. They can be found in Refs [1-6]. Whilst, Refs [1-5] were devoted to elastic buckling, Ref [6] examines the influence of boundary condition on buckling load of cones within the elastic-plastic range. However, the author was silent on the influence of such boundary condition on the deformed shape.

This paper seeks to investigate the influence of boundary condition on the buckling load and deformed shape of axially compressed cone.

## 2. METHODOLOGY

### 2.1 Numerical approach

For this research work, numerical analysis was conducted using ABAQUS finite element code. Consider a truncated cone with small radius,  $r_1$ , big radius,  $r_2$ , cone height,  $h$ , cone slant length,  $L$ , wall thickness,  $t$  and cone semi-vertex angle,  $\beta$ . Assume that cone is subjected to axial compression. The geometry parameters of analyzed cone are: radius of bigger

Table 2 Boundary conditions employed during numerical calculations.

		$u_x$	$u_y$	$u_z$	$\phi_x$	$\phi_y$	$\phi_z$
Case 1	Top	0	0	$\neq 0$	0	0	0
	Bottom	0	0	0	0	0	0
Case 2	Top	0	0	$\neq 0$	0	0	0
	Bottom	$\neq 0$	$\neq 0$	0	0	0	0

The specimens were modeled using four-node three-dimensional doubly curved shell elements with six degree of freedom (S4R). The material is modeled as elastic perfectly-plastic. Non-linear static analysis was carried out using the modified Riks method algorithm which is implemented in ABAQUS.

### 2.2 Experimentation

For this experiment, two nominally identical mild steel cones with the same nominal geometry as discussed in Section 2.1 were manufactured and tested. This would provide two experimental data values for cones with nominally the same geometry to ensure repeatability of experimental data. Cones were joined together using Tungsten Inert Gas (TIG) welding process. Conical specimens were subjected to axial compression using Universal Instron Machine. Incremental load was applied at the rate of 1.0 mm/min.

### 3. RESULTS AND DISCUSSION

Figure 1 depicts the comparison of deformed shape obtained from ABAQUS analysis of the two boundary conditions considered for relatively thick axially compressed conical shells. For case 1 (Figure 1a) where the big ends of the cone is clamped, it can be seen that there were axisymmetric bulging in the neighborhood of the small radius ends. Whereas, for case 2 (Figure 1b) where the big end of the cone allowed all displacement movement, it is apparent that as a result of change in boundary condition, the deformed shape also changes from bulging in the neighborhood of the small cone ends to a skirt like shape around the cone big ends. In addition, it was observed that the load carrying capacity of the cone reduces from 201.69 kN (for case 1) to 144.56 kN (for case 2) resulting in about 28% drop in the buckling load of the cone.

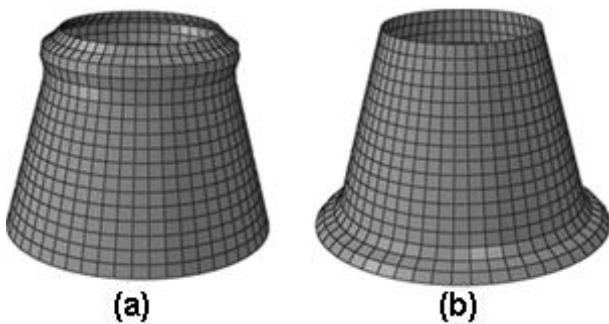


Figure 1 View of FE simulated deformed shape for different boundary conditions.

During the experiment, the axial shortening of the cones were recorded using the machine controller. Figure 2 depicts the plot of axial compressive force against axial shortening for both cones. It is apparent that both curves follow a similar pattern with both cones failing at nearly almost the same buckling load about 2% difference (163.68 kN vs 167.87 kN).

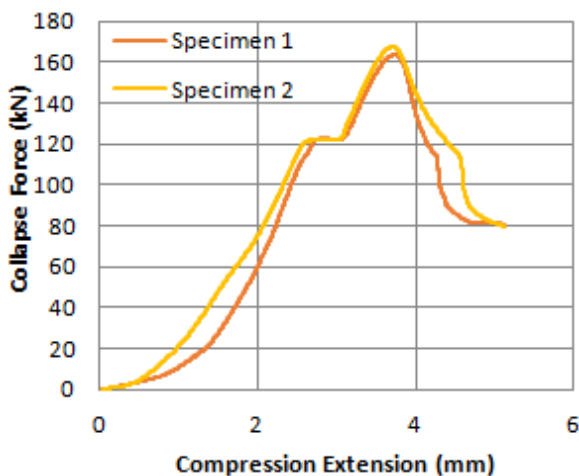


Figure 2 Plot of axial collapse force against compression extension for axially compressed cones.

To validate the deformed shape obtained from numerical simulation, it was decided to compare the experimentally obtained deformed shape with that

obtained from ABAQUS using the same boundary condition. Figure 3 depicts the comparison of deformed shape obtain from both experimental and numerical approach. It is evident that there is a good visual comparison of deformed geometries.

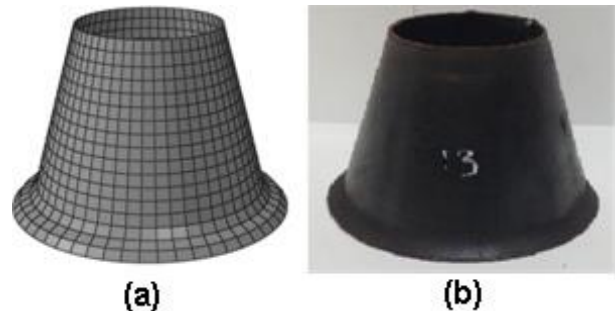


Figure 3 View of FE simulated deformed shape (Figure 3a) and collapsed cone (Figure 3b) subjected to axial compression.

### 4. CONCLUSION

Two nominally identical cones fail at nearly the same collapse load thereby confirming good repeatability of experiment data. Also, it can be concluded that change in boundary conditions has strong influence on both the buckling load and the deformed shape of relatively thick cones subjected to axial compression.

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