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# Mobility of rectangular plate with constraint and unconstraint edges

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**ABSTRACT** – Several theories have been established in calculating the mobility of rectangular plate. However, there is still a lacking of discussion on the mobility of plate with various boundary conditions. In this paper, modal summation approach is used to present the mobility of rectangular plates with various constraint and unconstraint edges to discuss their effect on the level of vibration across the frequency range. It is found that at very low frequency (1-20 Hz) that the more constraint the edges the lower the mobility.

## 1. INTRODUCTION

Concepts of mobility and impedance have been commonly used to investigate the responses of forced structures and power radiated or transmitted by the structures. Over the years, many technique and different approaches have been used to find the mobility of plates with various boundary conditions. The exact solutions available are for plate with simply supported, at least one pair of opposite edges. Extensive study has been carried out by Leissa [1] and presented the mode shape of the plates with various boundary conditions in monograph. Later, Warburton [2] produced a set of approximate frequency formulas derived from Rayleigh method in calculating mobility of the plates with various boundary conditions. The assumption is that the mode shape of the plate is the multiplication of vibrating beam eigenfunctions. The objective of this paper is to calculate and to present the mobility across the frequency of excitation with various boundary conditions using modal summation approach.

## 2. METHODOLOGY

Consider a plate with dimension $l_x$ and $l_y$ as shown in Figure 1. The reference point is located at the corner of the plate, $O$.

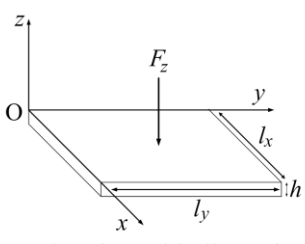


Figure 1 Rectangular plate with dimension $l_x$ and $l_y$ and thickness $h$.

According to Soedel [3], mobilities of finite plates can be written in terms of a modal summation. The mobility of the plate can be calculated by [3]

$$Y = \frac{v_z}{F_z} = j\omega \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\psi_{mn}(x_2, y_2)\psi_{mn}(x_1, y_1)}{\rho h l_x l_y [\omega_{mn}^2 (1 + j\eta) - \omega^2]} \quad (1)$$

where $v_z$ is the velocity of the plate in z-direction, $F_z$ is the force excitation, $\psi_{mn}(x_2, y_2)$ is the $(m,n)$ th bending natural mode and $\omega_{mn}$ is the $(m,n)$ th natural frequency and $\eta$ is the loss factor and $\omega$ is harmonic circular frequency with implicit time dependence $e^{j\omega t}$.

Mode shape of the plate $\psi_{mn}(x_2, y_2)$ is the product of the characteristic function of beam in *x*-direction and *y*-direction which is

$$\psi_{mn}(x, y) \cong \phi(x) \cdot \phi(y) \quad (2)$$

The characteristic function of the beam can be found in [3]. Natural frequencies for rectangular plates with various boundary conditions given by [2]

$$\omega_{mn} = \sqrt{\frac{Eh^2}{12\rho(1-v^2)} \cdot \left(\frac{\pi}{l_x}\right)^2 q_{mn}} \quad (3)$$

where

$$q_{mn} = \sqrt{G_x^4(m) + G_y^4(n)(a)^2 + 2(a)^2(b)} \quad \text{with}$$

$$a = l_x / l_y \quad \text{and} \quad b = vH_x(m)H_y(n) + (1-v)J_x(m)J_y(n)$$

for $v$ is Poisson ratio, $E$ is Young modulus, $h$ is thickness of the plate, and $\rho$ is density of the plate. The coefficient $G_x$, $G_y$, $J_x$, $J_y$, $H_x$ and $H_y$ for different boundary conditions correspond to the number of mode $(m,n)$ th can be found in [2].

## 3. RESULTS AND DISCUSSION

Results presented in the paper are calculated for aluminum plate with dimensions of $0.5 \times 0.6 \times 0.003$ m$^3$, Young modulus of $7.1 \times 10^{10}$ N/m$^2$, Poisson ratio of 0.3, and density of 2700 kg/m$^3$. Calculation in MATLAB was performed up to 1 kHz with frequency step of 0.005Hz. The notation of the plate boundary condition uses 'S' for simply supported, 'F' for free and 'C' for clamp.

### *3.1 Validation of Results*

The results of each plate with different boundary conditions are validated through finite element method (FEM) using software ABAQUS 6.10. A plate with the same dimension and material properties is created for simulation. The mesh resolution used is 0.01. The result calculated frequency range is up to 1 kHz. Figure 2 show mobility of F-F-F-F plate and C-C-C-C plate calculated from analytical model and ABAQUS 6.10. It can be seen that the result calculated from the analytical model shows good agreement with that compute from the ABAQUS 6.10.

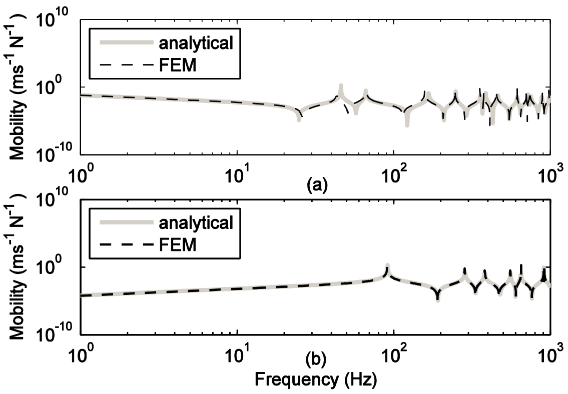


Figure 2 (a) Mobility of F-F-F-F (b) C-C-C-C plate with loss factor of $\eta = 0$ .

### *3.2 Mobility of Plates with All Edges Having Same Boundary Conditions*

Figure 3 shows the diagram of plate edges denoted by number. Plate with all edges having the same boundary conditions is indicated by edge-1 = edge-2 = edge-3 = edge-4 in Figure 3.

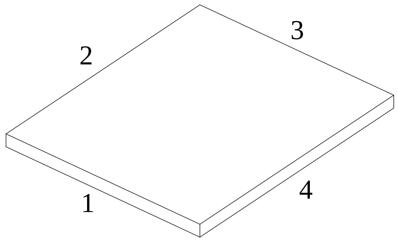


Figure 3 Rectangular plate with numbered edges.

Figure 4 show the mobility for a plate with uniform edges. It can be seen that plate with F-F-F-F edges has a different trend from frequency below 45 Hz compared to all other edges boundary condition. This is due to the even and rock mode. It also shows that F-F-F-F plate has the highest mobility and C-C-C-C plate has the lowest mobility at low frequency (1-20 Hz).

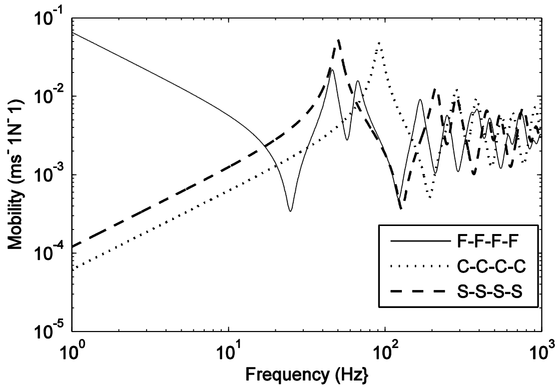


Figure 4 Mobility of plate with same boundary conditions at all edges with loss factor of $\eta = 0.1$.

### *3.3 Mobility of Plates with Identical Opposite Edges*

Identical opposite edges is when edge-1 = edge-2 and edge-3 = edge-4 referring to Figure 3. It can be seen in Figure 5 that the more the edges is constrained, the lower the mobility at low frequency (1-20 Hz). This can be seen for the C-C-S-S plate. Conversely, the plate with less constraint edges has the greatest mobility as seen for F-F-S-S plate at low frequency (1-20 Hz) in Figure 5.

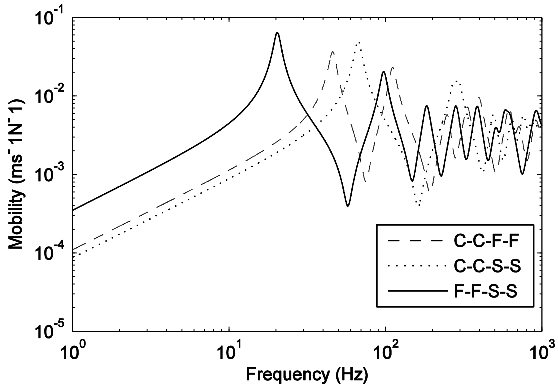


Figure 5 Mobility of plate with identical opposite side boundary condition with loss factor of $\eta = 0.1$.

## 4. CONCLUSIONS

The mobility of plates with various boundary conditions has been presented. Significant vibration level can be seen at low frequency (1-20 Hz) where the greatest mobility is given by the plate with less constraint edges. At high frequency, average level of mobility is almost similar for all the boundary conditions.

## 5. REFERENCES


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