Proceedings of Mechanical Engineering Research Day 2015, pp. 127-128, March 2015

On the dynamics of a beam-SDOF energy harvester system

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Keywords: Energy harvester, piezoelectric material, vibration

ABSTRACT – Dynamic model of a beam attached at the mass of a single-degree-of-freedom system is presented in this paper. The structural wave propagation method is used leading to solution of complex wave amplitudes across the beam. The proposed model is aimed at characterizing the behavior of the system to study the potential electrical energy harvested from such mechanism where the beam is a piezoelectric material.

1. INTRODUCTION

A conventional piezoelectric energy harvester is a system which consists of a cantilever beam structure. Numerous studies have been published regarding the mathematical modelling and the experimental tests of the electrical energy which can be harvested from this system [1-3].

This paper proposes the beam structure combined with a single-degree-of-freedom system (SDOF) to vibrate the beam from the motion of the SDOF system which is excited by an external harmonic force. Similar model has been by proposed by Wang et. al.[4] using the Hamilton's principle and Rayleigh-Ritz method. Here the structural wave propagation method is implemented where waves in the beam is assumed to consists of propagating flexural and near-field waves. Forces acting at the edge of the beam are determined according to the boundary conditions. The component of the complex wave amplitudes can be obtained through an inverse matrix process.

2. METHODOLOGY

Consider a uniform and thin beam with length L attached with the mass of a single-degree-of-freedom (SDOF) system as shown in Figure 1. The bending waves are assumed to propagate across the length of the beam. The wave amplitude in the beam is assumed for a time harmonic solution of the form [5]

$$u(x,t) = U(x)e^{j\omega t} \tag{1}$$

where the complex amplitude is given by

$$U(x) = a_1 e^{-jkx} + a_2 e^{jkx} + a_3 e^{-kx} + a_4 e^{kx} \tag{2}$$

where $a_{1,2}e^{\pm jkx}$ are the harmonic propagating waves and $a_{1,2}e^{\pm kx}$ are the near-field waves. The wavenumber is defined as

$$k = \sqrt{\omega}\left(\frac{\rho A}{EI}\right)^{\frac{1}{4}} \tag{3}$$

where E, I, ρ, A are respectively the Young Modulus, second mass moment of area, density and cross area of the beam.

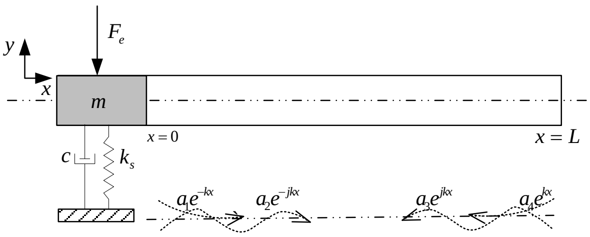

Figure 1 Beam-SDOF energy harvester system model.

The rotational displacement across the beam is defined as

$$\theta(x,t) = \frac{\partial u(x,t)}{\partial x} \tag{4}$$

Based on Eq. (2), the vector of the degrees of freedom is given can be given by

$$\begin{bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} e^{-jkx} & e^{jkx} & e^{-kx} & e^{kx} \\ -jke^{-jkx} & jke^{jkx} & -ke^{-kx} & ke^{kx} \\ e^{-jkL_x} & e^{jkL_x} & e^{-kL_x} & e^{kL_x} \\ -jke^{-jkL_x} & jke^{jkL_x} & -ke^{-kL_x} & ke^{kL_x} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{5}$$

Bending moment and shear force are given by

$$M(x) = EI\frac{\partial^2 u}{\partial x^2} \qquad F(x) = -EI\frac{\partial^3 u}{\partial x^3} \tag{6}$$

Substituting Eq. (5) into Eq. (6), the general matrix of the force equal to

$$\mathbf{F} = EI\,\mathbf{U}\tilde{\mathbf{a}} \tag{7}$$

where $\mathbf{F} = \begin{bmatrix} F_1 & M_1 & F_2 & M_2 \end{bmatrix}^{-1}$ and $\tilde{\mathbf{a}} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^{-1}$ with

$$\mathbf{U} = \begin{bmatrix} jk^3e^{-jkx} & -jk^3e^{-jkx} & -k^3e^{-kx} & k^3e^{kx} \\ -k^{2-jkx} & -k^2e^{jkx} & k^2e^{-kx} & k^2e^{kx} \\ jk^3e^{-jkL_x} & -jk^3e^{-jkL_x} & -k^3e^{-kL_x} & k^3e^{kL_x} \\ -k^{2-jkL_x} & -k^2e^{jkL_x} & k^2e^{-kL_x} & k^2e^{kL_x} \end{bmatrix}$$

From Figure 1, at $x = 0$, the shear force at the end of the beam equals to the net force acting on the SDOF system. If the SDOF system is excited with an unit external force $F_e = 1$, the boundary conditions can therefore be written as

$$\begin{aligned} F_1(0) &= (-\omega^2 m + j\omega c + k)\,\mathrm{U}(0) - F_e = F_S - F_e \\ M_1(0) &= 0 \\ F_2(L) &= 0 \\ M_2(L) &= 0 \end{aligned} \tag{8}$$

Equation (7) can then be re-arranged for $x = 0$ to give

$$\mathbf{U} = \begin{bmatrix} jk^3 - F_S & -jk^3 - F_S & -k^3 - F_S & k^3 - F_S \\ -k^2 & -k^2 & k^2 & k^2 \\ jk^3e^{-jkL} & -jk^3e^{-jkL} & -k^3e^{-kL} & k^3e^{kL} \\ -k^{2-jkL} & -k^2e^{jkL} & k^2e^{-kL} & k^2e^{kL} \end{bmatrix} \tag{9}$$

where $\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^{-1}$. The amplitude can then be calculated as

$$\tilde{\mathbf{a}} = \frac{\tilde{\mathbf{U}}^{-1}\mathbf{F}}{EI} \tag{10}$$

The results from Eq. (10) can be substituted back to Eq. (2) to obtain the vibration amplitude of the beam.

3. RESULTS DAN DISCUSSION

Simulation is conducted with parameters of the piezoelectric material (PZT) beam and the SDOF system given in Table 1.

Figure 2 shows the response for different mass of SDOF system at location of $x = 0.4L$. It can be seen that there are several significant resonances of the beam which can be a potential source of frequency excitation for harvesting the electrical energy.

4. CONCLUSIONS

Mathematical modeling of the dynamics of a beam-SDOF energy harvester system has been proposed using the wave propagation technique. For future study, it is of interest to investigate the case for vibration energy coming from the base which is more realistic in application where such energy harvester can be used to directly harvest the energy from a vibrating host structure.

Table 1 Parameters of the beam used for simulation

Properties	**Data**
Moment of area, I	5.475×10^{-13} m^4
Young's modulus, E	69×10^9 N/m^2
Density, ρ	7500 kg/m^3
Mass, m	0.1 kg
Spring stiffness, k_s	100 N/s
Damping constant, c	0.5 kg/s

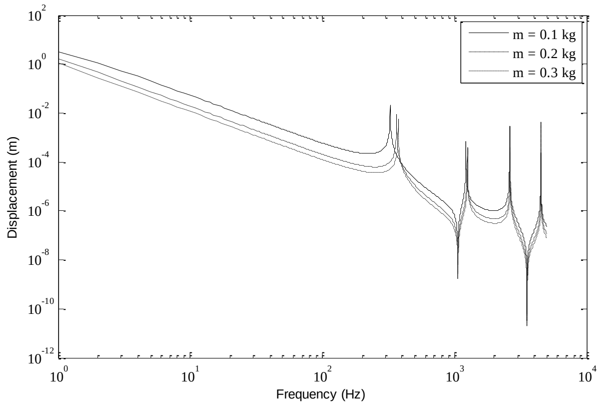

Figure 4 Displacement of the beam at $x = 0.4L$.

5. ACKNOWLEDGMENT

This research is conducted under The Research Acculturation Collaborative Effort Grant Scheme FRGS(RACE)/2012/FKM/TK01/02/1/F00146 from the Ministry of Higher Education Malaysia.

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