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Analysis on degree of nonlinearity in hardening nonlinear system of a vibration based energy harvesting device

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ABSTRACT – Analytical analysis is used to quantify the degree of nonlinearity in nonlinear hardening system in order to validate the usage of linear electromechanical coupling on weakly nonlinear hardening system. Approximate solution of harmonic balance method and multiple scales method is applied and both methods show the important effect of higher harmonics on the degree of nonlinearity.

1. INTRODUCTION

An energy harvesting device is modeled as a single degree of freedom mass-spring-damper system which are either mass excited or base excited where the energy harvested by the device is analogous to the energy dissipated by the damper.

The performance of this linear device is optimized when the natural frequency of the device is tuned to match the ambient frequency [1] and a slight mistune will bring a vast deterioration in the device performance. This thus creates a limitation on the linear generator since ambient frequency varies with time.

In order to overcome this limitation, researchers introduce nonlinearity into the system in the form of hardening spring. The theoretical study on the energy harvesting devices engaging a hardening spring has been keenly conducted by Quinn et al. [2] and Ramlan et al. [3]. A hardening spring system has a response that is less sensitive to the change in frequency which makes it fit for applications with varying ambient frequency. The response of the system with such a spring is able to widen the bandwidth to a much frequency range than the linear system, which provides some advantages in harvesting the vibration/mechanical energy from ambient sources since ambient frequency varies with time.

Currently, the degree of nonlinearity of devices employing a hardening spring is considered very weak. Hence, the electromechanical coupling for this weak nonlinear system is assumed to be similar to the one with a linear spring. A conclusive evident is seek theoretically to quantify the degree of nonlinearity in hardening system so that the validity of using linear electromechanical coupling on hardening nonlinear system can be established.

2. METHODOLOGY

Approximate solution of harmonic balance method and multiple scale method is pursued to quantify the degree of nonlinearity of a hardening system. The equation of motion for a base-excited hardening Duffing oscillator is given by [4]

$$m\ddot{s} + c\dot{s} + k_1 s + k_3 s^3 = -m\ddot{y} \quad (1)$$

where $s = x - y$ is the relative displacement between the seismic mass, x, and the housing, y, and $\mathrm{y} = \mathrm{Y}\cos(\omega t)$, k_1 is the linear spring constant and k_3 is the nonlinear spring constant with $k_3 > 0$ denoting a hardening system. Equation (1) can be expressed in non-dimensional form as

$$\ddot{u} + 2\zeta\dot{u} + u + \alpha u^3 = \Omega^2 \cos(\Omega\tau + \emptyset) \quad (2)$$

where $u = \frac{s}{Y}$, $\zeta = \frac{c}{2m\omega_n}$, $\alpha = \frac{k_3 Y^2}{k_1}$, $\Omega = \frac{\omega}{\omega_n}$, $\tau = \omega_n t$, $\emptyset =$ phase angle between excitation and response

2.1 Harmonic Balance Method

Single harmonic and two harmonic steady state solutions of equation (1) is seek using harmonic balance method as conversed by Hamdan and Burton [5]. The single-mode approximation for the steady state response is assumed to have the solution in the form of

$$u(\tau) = A \cos \Omega\tau \quad (3)$$

where A is the steady state response amplitude. In single-mode solution, only the fundamental harmonic approximation is obtained and the third harmonics is neglected.

Next, the two-mode approximation for the steady state response is assumed to have the solution in the form of

$$u(\tau) = A_1 \cos \Omega\tau + A_3 \cos 3\Omega\tau + B_3 \sin 3\Omega\tau \quad (4)$$

where A_1 is the first harmonic amplitude and A_3 and B_3 is the third harmonic amplitude. In two-mode solutions, the amplitude of first harmonic and third harmonic is acquired.

2.2 Multiple Scale Method

New independent variables is introduced to the Duffing oscillator in equation (2) according to

$$T_n = \varepsilon^n t \text{ for } n = 0, 1, 2, \dots \quad (5)$$

as shown in [6] and [7] where T_n is the independent time scales and t is a function of these time scales, $t = t(T_0, \dots T_n)$. This functional form allows the operation of differentiation which is the key step of this method to be expanded as follow

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \cdots, \quad (6a)$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 (2D_0 D_2 + D_1^2) + \cdots, \quad (6b)$$

where operators $D_n \equiv \frac{\partial}{\partial T_n}$, ε is small parameter.

Continuing with equating like powers of ε to zero and eliminating secular terms, the amplitude-frequency relation is obtained up to the second order.

3. RESULTS AND DISCUSSION

Frequency response for the Duffing oscillator shown in equation (2) obtained by using single-mode and two-mode harmonic balance method is presented in graph. Comparing single-mode frequency response graph with the two-mode frequency response graph, it is shown that the single-mode approximation did not reveal all of the essential features of the system where higher harmonics have an important effect on the resonance curves. This is presented in the graph where two-mode approximation graph is separated by an island that represents the nonlinear behavior of hardening system while single-mode graph does not.

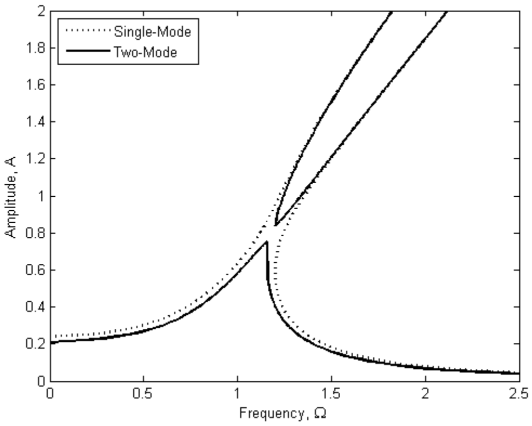

Figure 1 Frequency response obtained by using single-mode and two-mode harmonic balance method

The amplitude-frequency relation obtained for both first order and second order from the multiple scale method is also presented in graph. The relation from the graph shows a similar qualitative behavior with those obtained from harmonic balance method.

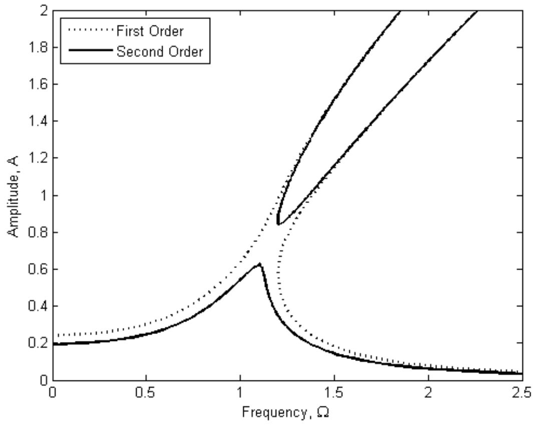

Figure 2 Frequency response obtained by using first order and second order of multiple scale method

4. CONCLUSIONS

Both harmonic balance method and multiple scales method are capable of showing the effect of higher harmonics on degree of nonlinearity of hardening nonlinear system when higher harmonics are taken into considerations. Therefore, the electromechanical coupling of hardening nonlinear system is not preferable to be assumed as similar with linear electromechanical coupling when only first harmonic is being taken into consideration.

5. ACKNOWLEDGEMENT

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