Variability of vibration input power to a beam structure

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ABSTRACT – Structure-borne source which transmits vibration power to the supporting structure especially in buildings plays a major role in contributing the noise pollution and this remains a challenging problem for noise treatment. In practice however, the lack of knowledge of phase of the excitation force from the structure-borne source creates variability in the input power. This paper discusses the quantification of the frequency-averaged mean and variance from the variability in the input power for the case of two excitation point forces to a beam structure. It is found that quantification of the frequency-average variability from a finite beam structure can be approached by using that from the corresponding infinite beam structure.

1. INTRODUCTION

Information of vibration input power from a structure-borne source is important as a preliminary control measure to allow a structural engineer to take preventive action by ensuring the supported structure is strong enough to absorb the potential vibration power. From this case, technique for structure-borne sound characterisation has been proposed [1,2]. Unfortunately, in order to obtain an accurate prediction of the injected input power, there still remains a problem due to the lack of some information for example the phase of the excitation force which creates uncertainties in the input power. This paper simulates the input power to a simple beam structure with harmonic excitation subjected to two forcing contact points similar to the previous work for plate structure [3]. This discusses the quantification of the frequency-averaged mean and variance of the variability in the input power.

2. FUNDAMENTAL EQUATION

2.1 Input Power in Finite Beam Structure

Figure 1 shows on a finite beam having length *a*, thickness *h* and width *b* subjected to two point forces. The input power can be expressed as:

$$P_{\text{in}} = \tfrac{1}{2}\,\text{Re}\left\{F^{H} Y F\right\} \qquad (1)$$

where $F = [F_1 e^{j\phi_1} \;\; F_2 e^{j\phi_2}]$ is the vector of the complex time-harmonic forces, Y is the mobility and ϕ is the phase.

In this paper, only translational force perpendicular to the receiver beam is taken into account. Therefore, the mobility matrix for two contact points is written as:

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \qquad (2)$$

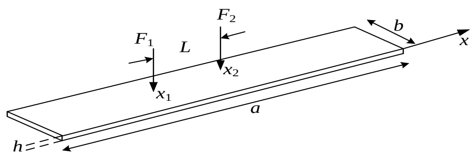


Figure 1 A finite beam with two point forces

where Y_{pq} is the point mobility for mode $p = q$ or transfer mobility for $p \neq q$. For excitation point at x_0 and x, the mobility at frequency ω is given by:

$$Y(x, x_0) = j\omega \sum_{n=1}^{\infty} \frac{\Phi_n(x_0)\Phi_n(x)}{\omega_n^2(1 + j\eta) - \omega^2} \qquad (3)$$

where $\Phi_n(x) = (2/M)^{1/2} \sin(n\pi x/a)^2$ is the n-th mass normalised mode with M is the total mass of the beam and $\omega_n = (B/m')^{1/2}(n\pi/a)^2$ is the natural frequency with the bending stiffness of beam $B = Ebh^3/12$, Young's modulus E, mass per unit length m' and the damping loss factor η. The total input power in Eq. (1) in terms of input mobility Y_p is therefore written as:

$$P_{\text{in}} = \tfrac{1}{2}\text{Re}\{Y_{11}\}F_1^2 + \tfrac{1}{2}\text{Re}\{Y_{22}\}F_2^2 + \text{Re}\{Y_{12}\}F_1 F_2 \cos\varphi \qquad (4)$$

where $\varphi = \phi_1 - \phi_2$ is the relative phase and note that the transfer mobility $Y_t = Y_{12} = Y_{21}$. For random excitation phase, the probability of relative phase $\varphi_i = 1/2\pi$ is assumed equal and constant. Thus, all the forces can also be assumed to have equal amplitudes. The mean and variance are therefore respectively given by:

$$\mu P_{\text{in}} = \tfrac{1}{2}\text{Re}\{\tilde{Y}_{11}\}F^2 + \tfrac{1}{2}\text{Re}\{\tilde{Y}_{22}\}F^2 \qquad (5)$$

$$\sigma^2 P_{\text{in}} = \tfrac{1}{2}\left(\text{Re}\{\tilde{Y}_t\}F^2\right)^2 \qquad (6)$$

2.2 *Input Power in Infinite Beam Structure*

The total input power for an infinite beam subjected to two point forces is also given the same as in Eq. (4). The input and transfer mobilities are given by:

$$Y_{\mathrm{p}} = \frac{\omega}{4Bk^3}(1-j) \qquad (7)$$

$$Y_{\mathrm{t}} = \begin{cases} \dfrac{j\omega}{4Bk^3}(-je^{-jkL} - e^{-kL}), & x \geq 0 \\ \dfrac{j\omega}{4Bk^3}(-je^{jkL} - e^{kL}), & x < 0 \end{cases} \qquad (8)$$

where $k = \sqrt[4]{m'\omega^2 / B}$. The mean and variance are also the same as in Equations (5) and (6).

2.3 *Averaging Over Frequency Bands*

The input power can be averaged over the frequency band defined as

$$P_{\mathrm{in}} = \frac{1}{\omega_1 - \omega_2}\int_{\omega_1}^{\omega_2} P_{\mathrm{in}}(\omega)d\omega \qquad (9)$$

where excitation frequency lies between two frequencies ω_1 and ω_2. Thus, the frequency-averaged input power in Eqs. (5) and (6) can be calculated numerically using Eq. (9). The relative standard deviation is therefore given by:

$$\gamma_{\sigma} = \frac{\langle \sigma P_{\mathrm{in}} \rangle}{\langle \mu P_{\mathrm{in}} \rangle} \qquad (10)$$

3. RESULTS AND DISCUSSION

Figure 2 shows the frequency-averaged mean power and standard deviation which are averaged over all possible excitation phases and with respect to the non-dimensional force separation distance *kL*. Note that the mean and standard deviation are normalised by the input power to an infinite beam which is also subjected to two point forces. The mean power and standard deviation are seen decay close to the mean power of the infinite beam.

For the same force separation distance *kL*, Figure 3 shows the relative standard deviation between the finite beam and infinite beams which are calculated numerically using Eq. (10) for two different damping loss factors. It can be seen that the result of relative standard deviation from the infinite beam follows the trend of that from the finite beam. Smaller damping gives a better agreement.

4. CONCLUSIONS

The variability of the input power has been modelled for the finite and infinite beam structures for two contact point forces. The quantification of the frequency-averaged mean and the standard deviation represented by the relative standard deviation shows a good agreement for both finite and infinite cases. It is found that quantification of the frequency-average variability from a finite receiver can thus be simply approached by using that from the corresponding infinite structure.

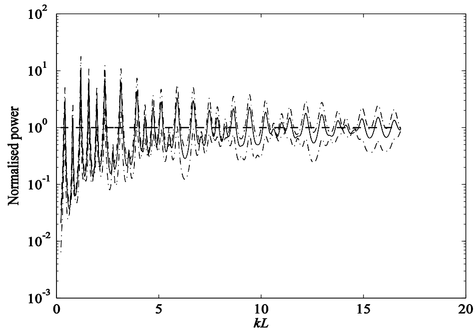
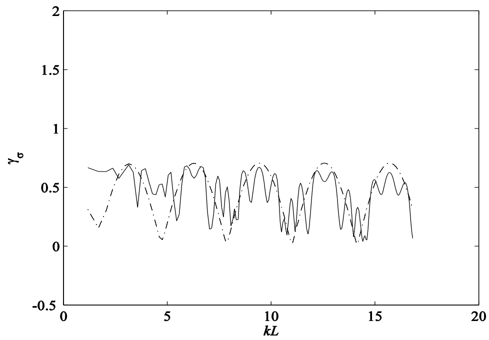
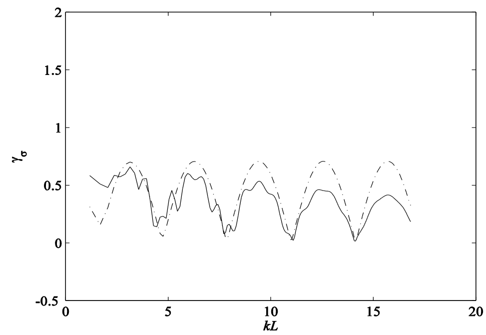


Figure 2 The normalised input power subjected to two harmonic point forces averaged over all possible excitation phases: (—) actual mean and (– - –) actual mean±standard deviation.



(a)



(b)

Figure 3 The relative standard deviation of input power averaged over frequency bands and phases: (—) finite beam and (– - –) infinite beam; (a) η = 0.05 and (b) η = 0.15.

4. REFERENCES

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